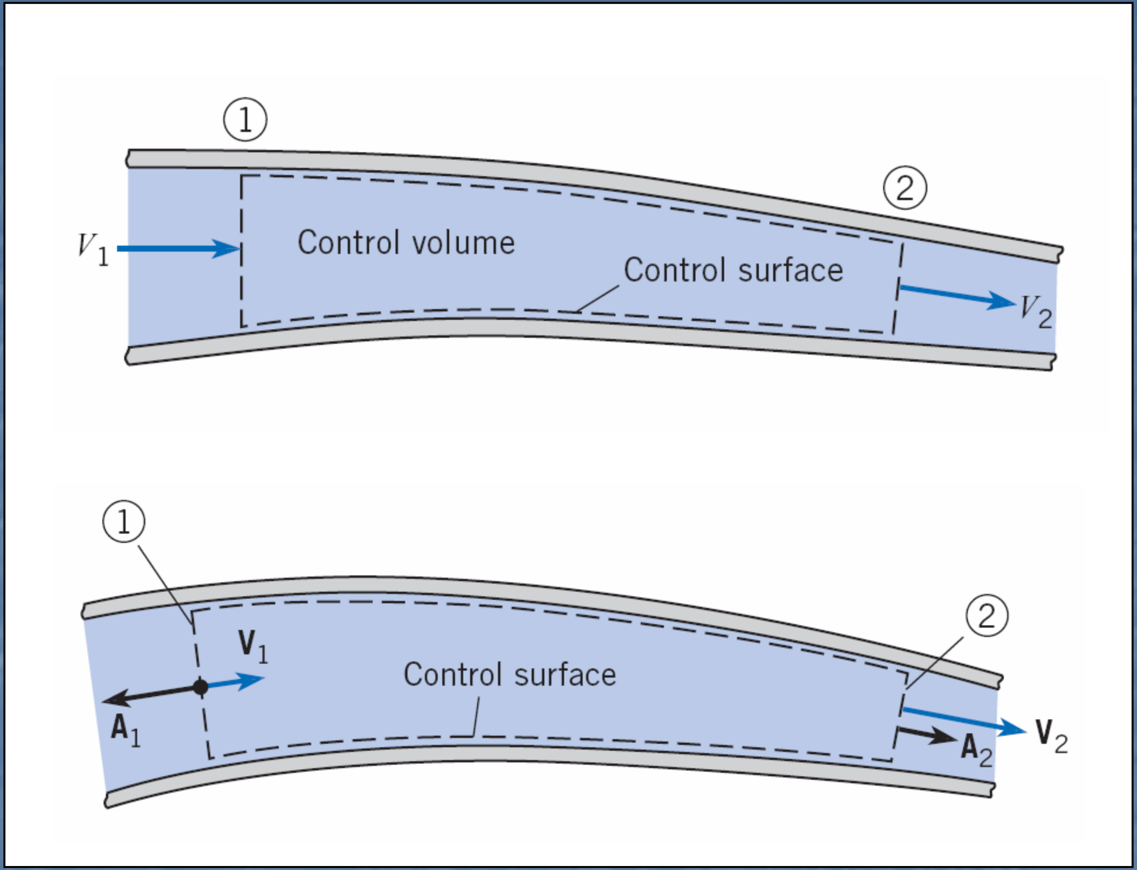
Property Transport across the Control Surface

$$\dot{m}_{net} = \dot{m}_{out} - \dot{m}_{in} = \dot{m}_2 - \dot{m}_1$$

$$= (\rho A V)_2 - (\rho A V)_1$$



In vector form

$$\dot{m}_{net} = (\rho A \bullet V)_2 + (\rho A \bullet V)_1$$

$$= \sum_{cs} \rho A \bullet V$$

Equation above states

If we sum the dot product $(\rho V \bullet A)$ for all flows into and out of the control volume, we get the net mass flow rate of the control volume

$$If \sum_{CS} \rho V \bullet A > 0 \;\; i.e\; Outflow(\dot{m}_{out}) > Inflow(\dot{m}_{in}), \quad \dot{m}_{net} > 0$$

$$If \sum_{CS} \rho V \bullet A < 0 \;\; i.e\; Outflow(\dot{m}_{out}) < Inflow(\dot{m}_{in}), \quad \dot{m}_{net} < 0$$

The net flow rate of an extensive property (B) out of the control volume is given by:

$$\dot{B}_{net} = \sum_{CS} b(\rho V \bullet A)$$

$$Where: \rho V \bullet A = \dot{m}$$

Velocity is Constant

A more general expression of the net flow rate of the extensive property (B) out of the control volume is given by:

$$\dot{B}_{net} = \int_{CS} b(\rho V \bullet dA)$$

Velocity is variable

INTENSIVE AND EXTENSIVE PROPERTIES

Extensive Properties: is Mass Dependent .
Intensive Properties: is Mass Independent

(B) is a Generic Extensive Property
(b) is the corresponding Generic Intensive Property,

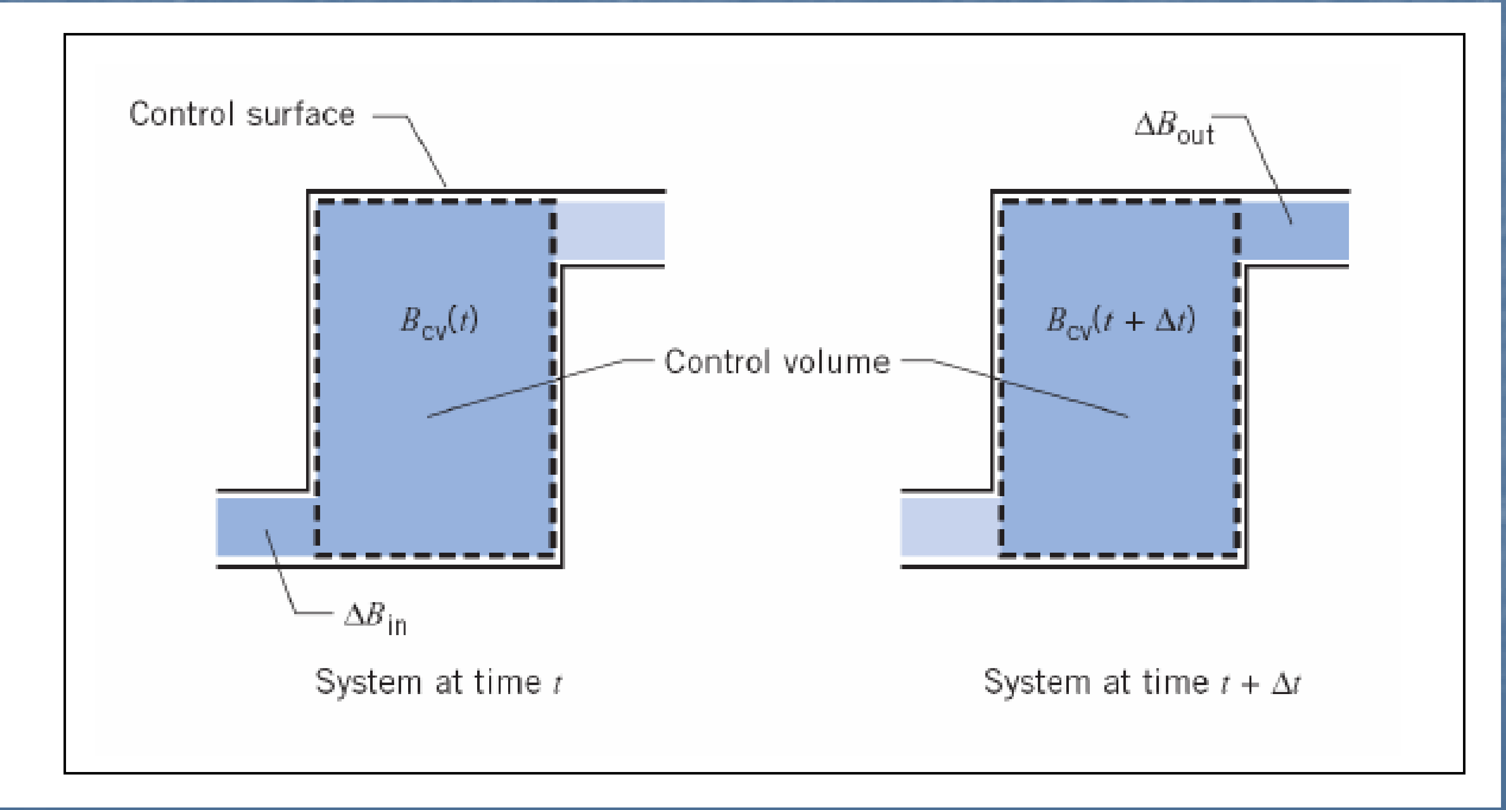
$$b = \frac{B}{M}$$

The amount of Generic Extensive Property (B) contained in a control volume at a given instant is

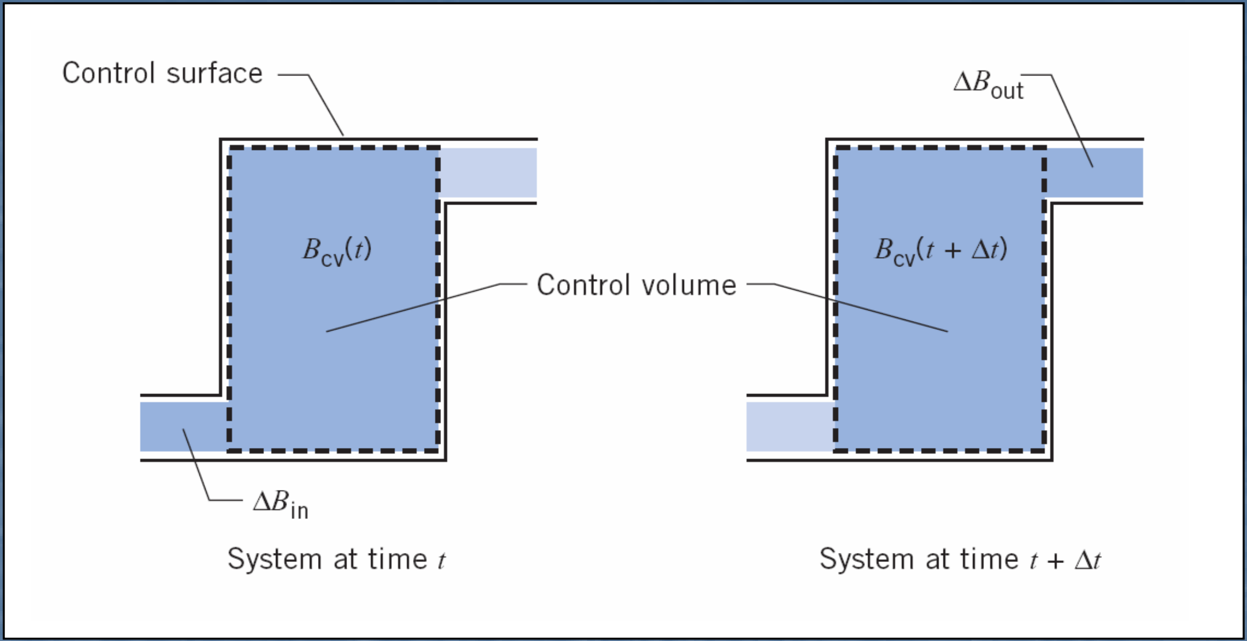
$$B_{CV} = \int_{CV} b\,dm = \int_{CV} b\rho\, dV$$

INTENSIVE AND EXTENSIVE PROPERTIES

Reynolds Transport Theorem involves the change of fluid properties inside the control volume and the flow properties across the control surface



REYNOLDS TRANSPORT THEOREM

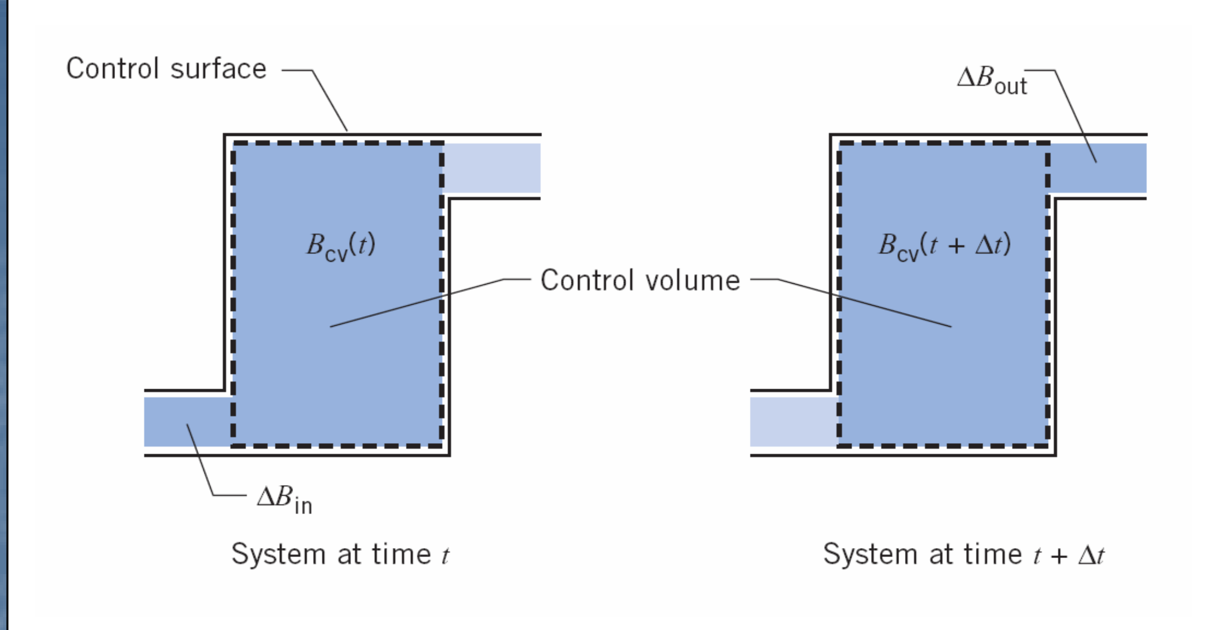


The property (B) of the system at time (t)

$$B_{sys}(t) = B_{cv}(t) + \Delta B_{in}$$

The property (B) of the system at time $(t + \Delta t)$

$$B_{sys}(t + \Delta t) = B_{cv}(t + \Delta t) + \Delta B_{out}$$



The rate of change of property (B)

$$\frac{dB_{sys}}{dt} = d\dot{B}_{cv} + \dot{B}_{out} - \dot{B}_{in}$$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\dot{B}_{cv} + \dot{B}_{net}$$

But $b = \dfrac{B}{mass}$

i.e. $$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho dQ + \int_{cs} b\rho V \bullet dA$$

REYNOLDS TRANSPORT THEOREM

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho dQ + \int_{cs} b\rho V \bullet dA$$

Equation above is called the Control Volume Equation

For different ports of inlets and outlets, the above equation can be written

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho dQ + \sum_{cs} b\rho V \bullet A$$

<u>In terms of mass flow rate</u>

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho dQ + \sum_{cs} \dot{m}_{out} b_{out} - \sum_{cs} \dot{m}_{in} b_{in}$$

SELECTION OF CONTROL VOLUME

For different ports of inlets and outlets

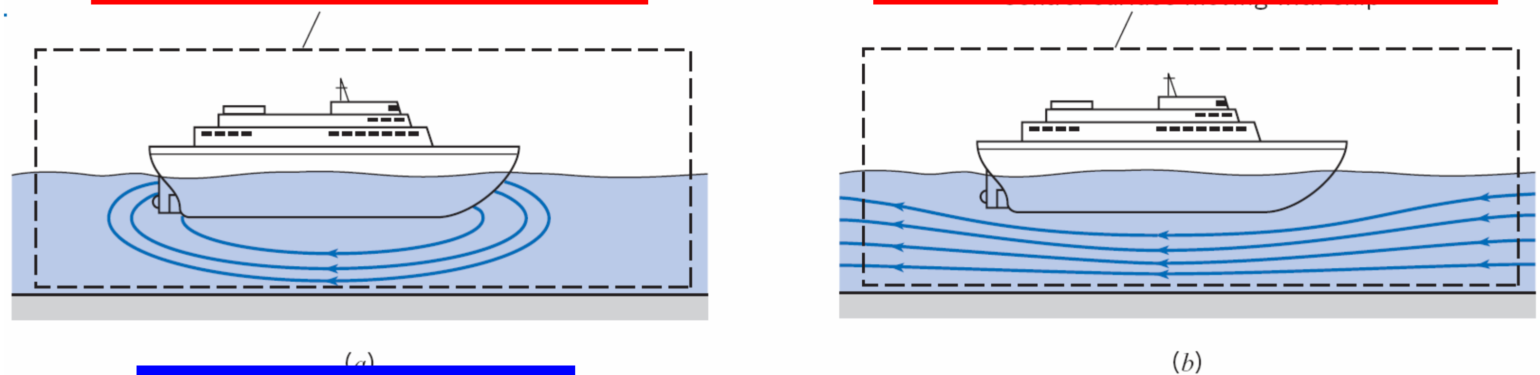
$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho dQ + \sum_{cs} b\rho V \bullet A$$

In terms of mass flow rate

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} b\rho dQ + \sum_{cs} \dot{m}_{out} b_{out} - \sum_{cs} \dot{m}_{in} b_{in}$$

Fixed Control Surface

Variable Control Surface



Unsteady Flow

Steady Flow

Case (a): $\frac{dB_{cv}}{dt} \neq 0$

Case (b): $\frac{dB_{cv}}{dt} = 0$

CONTROL VOLUME APPROACH

END OF LECTURE (3)